

DERIVING THE QGP-HADRON TRANSITION CURVE FROM TWO SEPARATE PARTITION FUNCTIONS

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Abstract

A method is developed to consistently satisfy the Gibbs equilibrium conditions between the quark-gluon and hadronic phase although each phase has been formulated in separate grand canonical partition function containing three quark flavours. The sector in the space of thermodynamic variables where the transition takes place is restricted to a curve, according to the phase diagram of QCD. The conservation laws of quantum numbers are also imposed on the transition curve. The effect of the inclusion of the newly discovered pentaquark states is considered. The freeze-out conditions of $S + S$, $S + Ag$ (SPS) and $Au + Au$ (RHIC) are found compatible with a primordial QGP phase, but the conditions indicated by $Pb + Pb$ (SPS) are not.

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1. Introduction

Quantum Chromodynamics is universally accepted as the theory of strong interactions. Within the context of this theory the phase of Quark-Gluon plasma receives accurate description. However, the formation of the hadronic phase, which is the final state of any possible primordial QGP state, still remains an open problem in view of QCD. On the other hand the hadronic multiplicities emerging from heavy-ion collisions have been extensively and successfully predicted by statistical models using a handful of thermodynamical parameters [1-8]. So the use of two separate models for the QGP and the hadron phase, called Hadron Gas (HG), offers a complementary approach.

QCD predicts that the transition between QGP and hadronic phase is of first order at high baryon densities (depicted by a critical line on the (T, μ_B) plane), while it is of higher order at small or zero baryon densities (crossover). The end point of the first order transition line is a critical point [9]. The transition points must be restricted to a curve on the phase diagram of temperature and baryon chemical potential. In view of this aspect any models to be used for the description of QGP and HG have to be matched properly at the transition between the two phases.

The aim of this work is to trace the sector of the space of thermodynamic variables where the QGP-hadron transition occurs, with the following requirements: a) Any mixed phase formed in the first order part of the transition must occupy only a curve in the space of the thermodynamic variables. This requirement is even more strong in the crossover area where a mixed phase does not exist. b) The Gibbs equilibrium conditions have to be satisfied, which amount to $T_{QGP} = T_{HG}$ for thermal equilibrium, $P_{QGP} = P_{HG}$ for mechanical equilibrium and $\{\mu\}_{QGP} = \{\mu\}_{HG}$ for chemical equilibrium, where $\{\mu\}$ stands for the set of chemical potentials used in the description of the two phases. c) All the conservation laws of quantum numbers like baryon number B , electric charge Q , strangeness S , etc. have to be satisfied at every point on the transition line, in a way that they could be extended for every number of flavours that are present to the system.

These problems are confronted every time separate partition functions are used for the two phases, but the simultaneous fulfilment of the above conditions has not been achieved. Among the numerous examples that exist, in [10], where only light, identical quarks are used

($u = d \equiv q$), the curve of equal pressures are made to approximately coincide by a choice on the external parameters B and a_s , something which does not allow matching when other flavours are introduced. In [11] the strange fugacity λ_s is discontinuous at the HG-QGP transition and the conservation of baryon number can only be accommodated at the case of first order transition. In [12] the strange chemical potential μ_s is also discontinuous. In [13] only q quarks are considered and the requirement of continuity of chemical potentials and conservation of baryon number leads to a mixed phase which occupies a surface and not a line on the (T, μ_B) plane. The same is true in [14-16] where also s quarks are included. In [17] there is an analogous situation as in [13] with a critical line at the (T, μ_B) plane but the conservation of baryon number is not considered. In [18] the q and s quark chemical potentials are continuous but baryon number and strangeness of the system are not kept constant during hadronisation, since hadrons evaporate from QGP. The considerations of [11-18] are consistent with a first order transition but cannot be valid at the crossover region. In this work all the thermodynamic variables and the pressure will be kept continuous at the transition line (in contrast with [10-12]), the first order part of the transition will be presented by a line on the (T, μ_B) plane (differing from [13-16]), in the mixed phase the quantum numbers will be conserved to each constituent phase (differing from [14]) and no evaporation of hadrons will be assumed from the system (differing from [18]).

Let us consider the requirements that a system with n_f quark flavours has to satisfy. Every conservation law accounts for two equations to be fulfilled. One sets the value of the quantum quantity, e.g. $\langle B \rangle_{QGP} = b_i$ and the other assures the conservation at the phase transition, e.g. $\langle B \rangle_{QGP} = \langle B \rangle_{HG}$. The total number of equations that must hold are, thus, $2n_f + 1$ (the unit accounts for the equality of pressures). Assuming full chemical equilibrium every quark flavour introduces one extra fugacity in the set of the thermodynamical variables, which, with the inclusion of volume and temperature, amount to $n_f + 2$. At the crossover region, the surviving free parameters to fulfil the necessary equations decrease to $n_f + 1$, since the equality of volume $V_{QGP} = V_{HG}$ results to the equality of densities between the two phases. At the first order transition line the free parameters are $n_f + 2$, since now $V_{QGP} \neq V_{HG}$. It is evident then that the necessary $2n_f + 1$ conditions can be fulfilled only at the first-order part of the transition and only when there is one flavour present, $n_f = 1$, or

when the u and d quarks are considered identical (q quarks, described by a single chemical potential μ_q). It has to be clarified that the conditions like $\langle B \rangle_{QGP} = b_i$ have to be satisfied in order to have a whole line of transition points. If these equations are dropped then we are left with $n_f + 1$ equations, which can be solved, but result to a unique point in the space of the thermodynamical parameters.

2. Expanding the fugacity sector

It is clear that in order to satisfy $2n_f + 1$ relations, every flavour has to be accompanied by two fugacities instead of one. The multiplicity data emerging from heavy ion collisions suggest that the thermalised hadronic system has not achieved full chemical equilibrium. First the strangeness partial chemical equilibrium factor γ_s had been introduced [2] and used extensively to model the data [3-4]. Also a similar factor for the light quarks γ_q was introduced [5] and used in many analyses [6]. Here the light u and d quarks will be accompanied by separate fugacities γ_u, γ_d . A factor γ_j controls the quark density $n_j + n_{\bar{j}}$ in contrast to the usual fugacity λ_j which controls the net quark density $n_j - n_{\bar{j}}$ [3]. These additional fugacities can serve the purpose of satisfying the necessary equations at the transition point, as well as, preserving the continuity of chemical potentials between the two phases.

A system with 3 flavours (u, d and s quarks) is described by the set of thermodynamical variables $(T, \lambda_u, \gamma_u, \lambda_d, \gamma_d, \lambda_s, \gamma_s) \equiv (T, \{\lambda, \gamma\})$. Assuming strangeness neutrality at the QGP phase leads to $\lambda_s = 1$. Setting $x = V_{HG}/V_{QGP}$, the set of equations to be satisfied at every phase transition point will be

$$P_{QGP}(T, \{\lambda, \gamma\}_{\lambda_s=1}) = P_{HG}(T, \{\lambda, \gamma\}_{\lambda_s=1}) \quad (1)$$

$$\langle n_B \rangle_{QGP}(T, \{\lambda, \gamma\}_{\lambda_s=1}) = x \langle n_B \rangle_{HG}(T, \{\lambda, \gamma\}_{\lambda_s=1}) \quad (2)$$

$$\langle n_B \rangle_{QGP}(T, \{\lambda, \gamma\}_{\lambda_s=1}) = 2\beta \langle n_Q \rangle_{QGP}(T, \{\lambda, \gamma\}_{\lambda_s=1}) \quad (3)$$

$$\langle n_Q \rangle_{QGP}(T, \{\lambda, \gamma\}_{\lambda_s=1}) = x \langle n_Q \rangle_{HG}(T, \{\lambda, \gamma\}_{\lambda_s=1}) \quad (4)$$

$$\langle n_S \rangle_{HG}(T, \{\lambda, \gamma\}_{\lambda_s=1}) = 0, \quad (5)$$

where n denotes densities. For isospin symmetric systems one has to set $\beta = 1$ in (3). Eqs. (1)-(5) have only one free variable, necessary to produce a whole transition line in the

phase diagram. At crossover $x = 1$, whereas at the first order transition line the inequality $V_{QGP} \neq V_{HG}$ preserves the survival of x as an extra variable.

3. A solution for the transition curve

Two simple models, will now be employed to apply the above considerations, the whole approach is, though, applicable to every partition function used. For the Hadron Gas phase all the hadronic states containing u , d and s quarks will be used [19]. The Bose-Einstein or Fermi-Dirac statistics applicable to each hadron will also be taken into account. Thus the HG partition function for point particles can be written down as

$$\ln Z_{HG\ pt}(V, T, \{\lambda, \gamma\}) = \frac{V}{6\pi^2 T} \sum_a \sum_i g_{ai} \int_0^\infty \frac{p^4}{\sqrt{p^2 + m_{ai}^2}} \frac{1}{e^{\sqrt{p^2 + m_{ai}^2}/T} \lambda_a^{-1} + \alpha} dp, \quad (6)$$

where g_{ai} are degeneracy factors due to spin and isospin and $\alpha = -1(1)$ for bosons (fermions). The index a runs over all hadronic families, each of which contains members with the same quark content and i to all the particles of this family. The fugacity $\lambda_a = \prod_j \lambda_j^{N_j - N_{\bar{j}}} \gamma_j^{N_j + N_{\bar{j}}}$, where $j = u, d, s$ and $N_j(N_{\bar{j}})$ is the number of $j(\bar{j})$ quarks contained in a hadron belonging to family a .

The repulsion which is due to the finite hadron size can be taken into account. Assuming that each hadrons' volume is proportional to its mass, $V_{ai}/m_{ai} = V_0$ the following factor can be calculated

$$f = \sum_a \sum_i n_{ai} V_{ai} = \frac{V_0}{6\pi^2 T} \sum_a \sum_i g_{ai} m_{ai} \int_0^\infty \frac{p^4}{\sqrt{p^2 + m_{ai}^2}} \frac{1}{e^{\sqrt{p^2 + m_{ai}^2}/T} \lambda_a^{-1} + \alpha} dp, \quad (7)$$

with V_0 remaining an open parameter controlling the hadron size. The pressure and the densities are then evaluated by

$$P_{HG} = \frac{P_{HG\ pt}}{1 + f} \quad n_{i\ HG} = \frac{n_{i\ HG\ pt}}{1 + f} \quad (8)$$

For the QGP phase a simple model containing 3 flavours is used. The quarks are non-interacting and only the presence of gluons is accounted for, as well as the effect of the vacuum through the MIT bag constant, B . A wealth of quark fugacities is easily accommodated, though, in this model. The QGP partition function is consequently

$$\ln Z_{QGP}(V, T, \{\lambda, \gamma\}) = \frac{N_s N_c V}{6\pi^2 T} \sum_j \int_0^\infty \frac{p^4}{\sqrt{p^2 + m_j^2}} \frac{1}{e^{\sqrt{p^2 + m_j^2}/T} (\lambda_j \gamma_j)^{-1} + 1} dp + V \frac{8\pi^2 T^3}{45} - \frac{BV}{T}, \quad (9)$$

where $N_s = 2$ and $N_c = 3$. The index j runs to all quarks and antiquarks and the fugacity $\lambda_{\bar{j}} = \lambda_j^{-1}$ and $\gamma_{\bar{j}} = \gamma_j$. The current quark masses are $m_u = 1.5$, $m_d = 6.75$ and $m_s = 117.5$ MeV [19].

Eqs. (1)-(5) are then solved for $x = 1$ for the crossover region. At the first-order QGP-HG transition a mixed phase is assumed

$$\ln Z_{mixed}(\delta, T, \{\lambda, \gamma\}) = \delta \ln Z_{QGP}(V_{QGP}, T, \{\lambda, \gamma\}) + (1 - \delta) \ln Z_{HG}(V_{HG}, T, \{\lambda, \gamma\}) , \quad (10)$$

where the set of fugacities $\{\lambda, \gamma\}$ are kept constant through out the first-order transition line, although the densities are not. The parameter $0 \leq \delta \leq 1$ and for $\delta = 0(\delta = 1)$ we have pure HG(QGP) phase. Eqs. (1)-(5) are solved for the pure phases leading to the determination of the thermodynamic parameters. The constraints are then automatically satisfied for every point in the mixed phase. For example eq. (2) is equivalent to $\langle B \rangle_{QGP}(V_{QGP}, T, \{\lambda, \gamma\}) = \langle B \rangle_{HG}(V_{HG}, T, \{\lambda, \gamma\})$. Thus $\langle B \rangle_{mixed} = \delta \langle B \rangle_{QGP} + (1 - \delta) \langle B \rangle_{HG} = \langle B \rangle_{QGP} = \langle B \rangle_{HG}$. The pressure of the mixed phase is also kept constant. Through the validity of eq. (1) $P_{mixed} = \delta P_{QGP} + (1 - \delta) P_{HG} = P_{QGP} = P_{HG}$.

The HG partition function (6)-(8) and the QGP partition function (9) is used to the system of eqs. (1)-(5). This system accepts as solution for the variable γ_s the value 0, since then eq. (5) is automatically satisfied. This a trivial solution because it is equivalent to the absence of the strange quarks in the system and so such solutions should be excluded. A non-trivial solution for the thermodynamic variables is depicted for the parameters $B^{1/4} = 280$ MeV and $V_0 = 1.6/(4B)$. The position of the critical point is set at $\mu_{B \text{ cr.p.}} = 360$ MeV as in [20]. For the ratio of the volumes $x = V_{HG}/V_{QGP}$ the adopted form for $\mu_B > \mu_{B \text{ cr.p.}}$ is $x = 1 + \left(\frac{\mu_B - \mu_{B \text{ cr.p.}}}{1000 - \mu_{B \text{ cr.p.}}} \right)^2 0.05$. It should be pointed that the position of the critical point and the expansion ratio in the 1st order transition cannot be predicted by the simple model used for these calculations, due to the absence of interaction from both phases. The temperature T is displayed as function of the baryon chemical potential μ_B in Fig. 1. The critical point is also depicted and it divides the crossover line (slashed line) from the first order transition line (solid line). The relative chemical equilibrium fugacity γ_u is displayed as function of μ_B in Fig. 2. This particular solution leads to the gradual suppression of γ_u as baryon chemical potential increases. The connection of γ_u and γ_d for isospin symmetric solution to both QGP

and HG phases is depicted in Fig. 3. The line $\gamma_u = \gamma_d$ is also drawn for comparison. The relative chemical equilibrium factor γ_s is drawn as function of baryon chemical potential in Fig. 4. The solution has a part with suppressed γ_s and another with enhanced γ_s , but the values are not far from unit.

One direct consequence of the simultaneous solution of eqs. (1)-(5) is that the relative chemical equilibrium fugacities have values that depend on each other at every transition point. This is easily realised in the condition $\langle S \rangle_{HG} = 0$ (for $\lambda_s = 1$). The solution of this condition is greatly simplified by the use of the Boltzmann approximation and the assumption that isospin symmetry leads to the approximate solution $\lambda_u = \lambda_d \equiv \lambda_q$ and $\gamma_u = \gamma_d \equiv \gamma_q$. Neglecting trivial solutions, where $\gamma_s = 0$, the zero strangeness condition can be solved to give

$$\gamma_s = \frac{F_K(T) - F_H(T)\gamma_q(\lambda_q + \lambda_q^{-1})}{2F_{\Xi}(T)}. \quad (11)$$

In eq. (11) F_K represents the Kaon mesons, F_H the Hyperon baryons (Λ 's and Σ 's) and F_{Ξ} the Ξ baryons and the summation

$$F_a(T) = \frac{T}{2\pi^2} \sum_i g_{ai} m_{ai}^2 K_2\left(\frac{m_{ai}}{T}\right) \quad (12)$$

includes all members of every family. In the above relation K denotes a modified Bessel function of the second kind. It is evident from eq. (11) that the increase of the relative chemical equilibrium factor for light quarks, γ_q and the increase of the light quark fugacity λ_q leads, at constant temperature, to the decrease of the factor γ_s and, thus, to the strange content of the system at the transition point.

4. Inclusion of pentaquarks

Recently there has been evidence of hadrons containing five quarks. These 5-quark states are the $\Theta^+(1540)$ [21] with $I = 0$ and quark content $uudd\bar{s}$ and the $\Xi^*(1862)$ with $I = 3/2$. The content of the states $\Xi^*(1862)$ is $ssdd\bar{u}$ (for the state with electric charge $Q=-2$), $ssud\bar{u}$ (with $Q=-1$), $ssudd$ (with $Q=0$) and $ssuud$ (with $Q=+1$). The existence of the first three of the states $\Xi^*(1862)$ has been confirmed [22]. Due to the quark content of these states the eqs. (1)-(5) are altered. This can easily be realised if the corresponding equation of (11) is

written down as

$$\gamma_s = \frac{F_K(T) + F_\Theta(T)\gamma_q^3(\lambda_q^2 + \lambda_q^{-2})(\lambda_q + \lambda_q^{-1}) - F_H(T)\gamma_q(\lambda_q + \lambda_q^{-1})}{2[F_\Xi(T) + F_{\Xi^*}(T)\gamma_q^2]}. \quad (13)$$

The existence of Θ hadron drives γ_s to higher values with a strong dependence on γ_q and λ_q , whereas the inclusion of the Ξ^* states contribute to decrease of γ_s .

The system of eqs. (1)-(5) is then solved with the inclusion of the $\Theta^+(1540)$ and $\Xi^*(1862)$ states¹ for the same parameters B , V_0 and the same volume ratio x as in the case without the inclusion of the pentaquarks. The results are shown in Figs. 1-4. It is evident that, now, at the transition line γ_s has increased compared to the case when the 5-quarks were neglected. Also, $\gamma_s > 1$, which is in agreement with the enhancement of strangeness production in the QGP phase.

5. Application to heavy-ion data

The quantum constraints previously discussed can be used as a diagnostic tool for a primordial QGP phase. Assuming (a) that a quark-gluon phase has been formed in a collision experiment and (b) that the chemical freeze-out occurs right after the transition to the hadronic phase, then the freeze-out thermodynamic variables have to fulfil eqs. (1)-(5). If, on the contrary, no quark-gluon state is formed before hadronization, then, the restriction on the freeze-out conditions of the system is diminished. The thermodynamic variables are extracted through a fit of the experimentally measured particle multiplicities or ratios to a statistical model. Such a technique has been successful. If now the additional constraints (1)-(5) are imposed, the question that arises is whether a successful fit is also produced or the variables that these constraints imply are inconsistent with the data.

It is easier for the fitting procedure to form a subset of the necessary equations and apply them in order to determine first a subset of the available parameters. Eq. (4), when eqs. (2) and (3) are valid, can equivalently be rewritten in the form

$$\langle n_B \rangle_{HG}(T, \{\lambda, \gamma\}_{\lambda_s=1}) = 2\beta \langle n_Q \rangle_{HG}(T, \{\lambda, \gamma\}_{\lambda_s=1}) \quad (14)$$

Eqs. (3), (5) and (14) (referred to as Set B) now form a set of equations that do not depend on the parameters V_0 for the particle size, B (MIT bag constant) nor the ratio x applicable

¹The partition function of eqs. (6)-(8) has been used for the HG phase and the partition function of eq. (9) for the QGP phase.

to the first order transition line. Especially eq. (5) is completely model independent and under the assumptions of isospin symmetry and Boltzmann statistics can acquire the simple form (11) or (13). Eq. (14) is also model independent since the volume corrections have been cancelled out. Eq. (3) depends only on the quark masses.

On the contrary, eqs. (1) and (2) are model dependent and contain unknown parameters. However, if the freeze-out parameters are determined they can be inserted to eq. (2) to determine V_0 (assuming that x is known) and then eq. (1) can be used to determine B . This task serves to show that eqs. (1) and (2) have a real solution and contributes to the overall consistency of the technique.

The constraints of set B have been used in the search for the freeze out parameters with data from the experiments $S+S$ [23], $S+Ag$ [24] (NA35) at beam energy 200 AGeV, $Pb+Pb$ [25] (NA49) at beam energy 158 AGeV and $Au+Au$ [26] (STAR) at $\sqrt{s_{NN}} = 130$ GeV. The data used are listed in Table 1 and they are in all cases full phase space multiplicities except from the RHIC data which are measured in the midrapidity. The experiments are so chosen because they do not produce great baryon chemical potential at freeze out and so they are probably at the crossover area [20], allowing one to set $x = 1$. The technique can be applied to the first order transition case, determining the freeze-out variables, but then the unimportant parameters V_0 and B cannot not be uniquely determined.

The theoretical calculation of the particle multiplicity necessary to perform a fit to the experimental data has been carried out with the partition function (6), (7). The right Bose-Einstein or Fermi-Dirac statistics for every particle has been used throughout the calculations. The feeds from the decay of resonances have also been included.

The results from the fits performed are listed in Table 2. The set of constraints B includes the conditions that the system freezes on the QGP-Hadron transition line. For comparison, the χ^2/dof result from the fit with only the constraints relevant to the hadronic phase (eqs. (5) and (14) with λ_s left as a free parameter) is also listed in the sector of set A. The value of β is set to 1 in the case of $S+S$, 1.1 in the case of $S+Ag$, 1.27 for $Pb+Pb$ and 1.25 for $Au+Au$. Two fits are performed in each case, one with all the multiplicities included and one without the multiplicity that contains the pions. The reason is that the inclusion of

this multiplicity deteriorates the quality of the fit [8,27]². So a bad fit, when the additional constraints of set B are imposed, may be partly due to the presence of this multiplicity. For this reason the fit without the pions is more reliable.

For the $S + S$ and $S + Ag$ data the quality of the fit with set B is of medium quality ($\chi^2/dof = 2.94$ and 1.91 , respectively) when the pions are present. This is not far worse, though, than in the case of set A. When the pions are excluded the fit turns out to be very good in the case of set B ($\chi^2/dof = 0.47$ and 0.0675 , respectively), proving these cases to be completely compatible with a primordial quark-gluon phase.

In the case of $Pb + Pb$ the imposition of set B severely worsens the quality of the fit. The situation cannot be remedied with the exclusion of pions and χ^2/dof remains at the value of 18. Also the fitted temperature in case of set B is unrealistically high.

The findings concerning the $S + S$ and $S + Ag$ data are in agreement with the proximity of the chemical freeze out points of these experiments to the Statistical Bootstrap critical line that was found in [8]. On the contrary the freeze out point of $Pb + Pb$ was not found to possess such an attribute in [29], also in agreement with the present results.

In the case of RHIC the imposition of set B does not lead to a good quality fit in the presence of pions ($\chi^2/dof = 3.86$), but the fit turns out to be quite good when the pions are excluded ($\chi^2/dof = 1.2$), so the thermodynamic parameters are compatible with a QGP phase.

The extracted parameters in case of set B are inserted to eqs. (1) and (2) and the parameters V_0 and $B^{1/4}$ are also determined. It is interesting that in the cases of $S + S$, $S + Ag$ and $Au + Au$ (without the pions), which have been proven compatible with set B, all the calculated values of V_0 and $B^{1/4}$ are close, compatible with a unique value for these parameters.

All the previous fits have been performed without the presence of the pentaquark states. Similar fits have also been performed with the inclusion of the pentaquarks. The difference in the extracted parameters, apart from the volume, is found to be at most 1.4% and in the parameter VT^3 at most 6% and so they are not listed.

²The presence of excess of pions, though, can be connected with a primordial high entropy phase or with a phase with the chiral symmetry restored [28].

The necessity of the expansion of the fugacity sector with the partial equilibrium fugacities is also revealed with the application of the present technique. If these fugacities are set to $\gamma_u = \gamma_d = \gamma_s = 1$ then the sector of the phase space that is compatible with the QGP-hadron transition is severely limited. In that case if a similar fit to the set B is performed, apart from the fact that eqs. (3) and (14) cannot be accommodated, the fit turns out to be worse. The result in the case without the pions is then $\chi^2/dof = 0.61, 1.05, 26.1$ and 1.86 for $S + S$, $S + Ag$, $Pb + Pb$ and $Au + Au$ respectively. In the case of $Au + Au$ the compatibility with the QGP phase turns out to be dubious now.

6. Conclusions

Although two different partition functions are used for the description of the quark and hadronic side of matter, it is possible to preserve the continuity of all chemical potentials and, of course, temperature at the transition between one another, which is confined on a curve. Also, all the constraints imposed by the conservation laws of quantum quantities and the continuity of pressure can be applied, leading, at the same time to a non-trivial solution of the thermodynamic variables into a three quark flavour system. The key issue for the success of this project is the expansion of the fugacity sector of the available variables and the, already, introduced relative chemical equilibrium variables can be used to serve that purpose.

The restrictions on the freeze-out conditions imposed by the existence of a quark-gluon state in the early stages after a collision experiment can be applied to every case that the thermalisation of the produced hadrons has been proven. They can serve to separate the experiments compatible with QGP state from those that are not. In a simplified and quick to use form these restrictions acquire the form of eqs. (11) or (13). The expansion of the fugacity sector with the partial equilibrium fugacities, though, magnifies the part of the phase space allowed by such constraints.

The whole methodology that was presented can be used for every grand canonical partition function adopted for the description of the HG or QGP phase. The inclusion of interaction is crucial for the prediction of the critical point and the volume expansion ratio, which could not be determined by the models used in this work. At the moment lattice

calculations have led to the determination of the accurate quark-gluon equation of state with three quark flavours at finite baryon chemical potential [20,30]. It would be interesting, though, if these calculations could be extended with the inclusion of the relative chemical equilibrium variables for light and strange quarks, allowing matching with the existing hadron gas models. For the hadronic side of matter the inclusion of the attractive part of interaction can be incorporated via the statistical bootstrap [7,8], where the prediction of a critical point is also possible [31]. The incorporation of the full set of parameters γ_i to these studies would allow for a more precise matching with a primordial quark phase.

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| $S + S$ | | $S + Ag$ | | $Pb + Pb$ | | $Au + Au$ | |
|-----------------|-----------------|-----------------|----------------|-----------------|-----------------|-------------------------|-------------------|
| K^+ | 12.5 ± 0.4 | K_s^0 | 15.5 ± 1.5 | N_p | 362 ± 5.1 | Λ | 17.20 ± 1.75 |
| K^- | 6.9 ± 0.4 | Λ | 15.2 ± 1.2 | K^+ | 103 ± 7.1 | $\bar{\Lambda}$ | 12.15 ± 1.25 |
| K_s^0 | 10.5 ± 1.7 | $\bar{\Lambda}$ | 2.6 ± 0.3 | K^- | 51.9 ± 3.6 | Ξ^- | 2.11 ± 0.23 |
| Λ | 9.4 ± 1.0 | \bar{p} | 2.0 ± 0.8 | K_s^0 | 81 ± 4 | Ξ^+ | 1.77 ± 0.19 |
| $\bar{\Lambda}$ | 2.2 ± 0.4 | $p - \bar{p}$ | 43 ± 3 | ϕ | 7.6 ± 1.1 | $\Omega + \bar{\Omega}$ | 0.585 ± 0.150 |
| \bar{p} | 1.15 ± 0.40 | $B - \bar{B}$ | 105 ± 12 | Λ | 53 ± 5 | p | 26.37 ± 2.60 |
| $p - \bar{p}$ | 21.2 ± 1.3 | $h^{(*)}$ | 186 ± 11 | $\bar{\Lambda}$ | 4.64 ± 0.32 | \bar{p} | 18.72 ± 1.90 |
| $B - \bar{B}$ | 54 ± 3 | | | Ξ^- | 4.45 ± 0.22 | K_s^0 | 36.7 ± 5.5 |
| $h^{(*)}$ | 98 ± 3 | | | Ξ^+ | 0.83 ± 0.04 | ϕ | 5.73 ± 0.78 |
| | | | | Ω | 0.62 ± 0.09 | K^{*0} | 10.0 ± 2.70 |
| | | | | $\bar{\Omega}$ | 0.20 ± 0.03 | $\pi^{(*)}$ | 239 ± 10.6 |
| | | | | $\pi^{(*)}$ | 619 ± 35.4 | $\pi^{(*)}$ | 239 ± 10.6 |
| | | | | $\pi^{(*)}$ | 639 ± 35.4 | K^+/K^- | 1.092 ± 0.023 |
| | | | | | | \bar{K}^{*0}/K^{*0} | 0.92 ± 0.27 |
| | | | | | | $\bar{\Omega}/\Omega$ | 0.95 ± 0.16 |

(*) This multiplicity has not been used in the fits where the pions are excluded.

Table 1. The full phase space multiplicities from the collision experiments $S + S$ (NA35), $S + Ag$ (NA35) and $Pb + Pb$ (NA49), as well as the midrapidity multiplicities and ratios from $Au + Au$ (STAR), used in the fits.

| | | $S + S$ | $S + Ag$ | $Pb + Pb$ | $Au + Au$ |
|-------------------|-----------------|-------------------|-------------------|---------------------|-------------------|
| fit with all | | | | | |
| set A | χ^2/dof | 4.03/3 | 3.50/1 | 16.4/7 | 6.01/9 |
| set B | χ^2/dof | 14.7/5 | 5.72/3 | 162/9 | 42.4/11 |
| | T (MeV) | 243 ± 16 | 275 ± 36 | 436.5 ± 7.5 | 345 ± 32 |
| | λ_u | 1.536 ± 0.022 | 1.613 ± 0.033 | 1.668 ± 0.020 | 1.082 ± 0.011 |
| | λ_d | 1.534 ± 0.022 | 1.638 ± 0.035 | 1.728 ± 0.022 | 1.086 ± 0.011 |
| | γ_u | 0.584 ± 0.076 | 0.458 ± 0.091 | 0.2730 ± 0.0044 | 0.349 ± 0.033 |
| | γ_d | 0.586 ± 0.077 | 0.472 ± 0.093 | 0.2937 ± 0.0049 | 0.381 ± 0.035 |
| | γ_s | 0.402 ± 0.059 | 0.309 ± 0.063 | 0.1880 ± 0.0026 | 0.322 ± 0.031 |
| | VT^3 | 152.0 ± 5.9 | 279.6 ± 8.5 | 679 ± 18 | 377 ± 26 |
| | $V_0^{(*)}$ | 5.4 ± 1.2 | 4.87 ± 0.96 | 2.332 ± 0.083 | 3.57 ± 0.32 |
| | $B^{1/4}$ (MeV) | 324 ± 23 | 358 ± 49 | 547.8 ± 9.5 | 442 ± 42 |
| fit without pions | | | | | |
| set A | χ^2/dof | 0.356/2 | 0/0 | 8.97/5 | 2.12/7 |
| set B | χ^2/dof | 1.88/4 | 0.135/2 | 128/7 | 10.8/9 |
| | T (MeV) | 194.9 ± 6.5 | 209 ± 19 | 443.6 ± 8.8 | 221 ± 24 |
| | λ_u | 1.605 ± 0.037 | 1.661 ± 0.043 | 1.746 ± 0.017 | 1.075 ± 0.011 |
| | λ_d | 1.599 ± 0.036 | 1.695 ± 0.046 | 1.817 ± 0.019 | 1.081 ± 0.011 |
| | γ_u | 0.949 ± 0.095 | 0.77 ± 0.18 | 0.2612 ± 0.0042 | 0.69 ± 0.18 |
| | γ_d | 0.958 ± 0.097 | 0.79 ± 0.18 | 0.2801 ± 0.0046 | 0.74 ± 0.19 |
| | γ_s | 0.85 ± 0.10 | 0.60 ± 0.16 | 0.1987 ± 0.0029 | 0.79 ± 0.23 |
| | VT^3 | 94.1 ± 9.5 | 199 ± 40 | 621 ± 16 | 272 ± 53 |
| | $V_0^{(*)}$ | 6.8 ± 1.4 | 6.2 ± 3.1 | 2.216 ± 0.093 | 5.4 ± 3.9 |
| | $B^{1/4}$ (MeV) | 279 ± 11 | 291 ± 29 | 556 ± 11 | 305 ± 37 |

(*) V_0 is measured in $(10^{-11} \text{ MeV}^{-4})$.

Table 2. The results of fits on the $S + S$ (NA35), $S + Ag$ (NA35), $Pb + Pb$ (NA49) and $Au + Au$ (STAR) data with the imposition of the set of constraints A and B, without the inclusion of the pentaquark states.

Figure Captions

Fig. 1 Temperature as function of the baryon chemical potential at the QGP-Hadron gas transition line, without and with the inclusion of the pentaquark states.

Fig. 2 Relative chemical equilibrium variable of u -quark, γ_u , as function of the baryon chemical potential at the QGP-Hadron gas transition line, without and with the inclusion of the pentaquark states.

Fig. 3 Relative chemical equilibrium variable of d -quark, γ_d , as function of relative chemical equilibrium variable of u -quark, γ_u , at the QGP-Hadron gas transition line, without and with the inclusion of the pentaquark states for the isospin symmetric case. The line $\gamma_d = \gamma_u$ is also depicted.

Fig. 4 Relative chemical equilibrium variable of s -quark, γ_s , as function of the baryon chemical potential at the QGP-Hadron gas transition line, without and with the inclusion of the pentaquark states.







